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Unsteady MHD Free Convective Flow past a Moving Vertical Plate with Time Dependent Suction and Chemical Reaction in a Slip Flow Regime

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Abstract

The objective of the present paper is to investigate chemical reaction effect on unsteady hydromagnetic free convection flow of a viscous, incompressible, electrically conducting fluid with heat and mass transfer past a moving porous vertical plate of infinite length with time dependent suction in the presence of heat source in a slip flow regime. Slip flow conditions for the velocity, jump in temperature and jump in concentration are taken into account in the boundary conditions. The governing equations involved in the present analysis are solved by perturbation technique. The velocity, temperature and concentration are studied for different parameters like Prandtl number, velocity ratio parameter, magnetic field parameter, chemical reaction parameter and slip parameter due to jump in concentration. It is inferred from the graphs that the temperature of the plate decreases with increasing the values of the Prandtl number. The concentration is decreased in the presence of chemical reaction and slip parameter due to jump in concentration.

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Keywords: MHD; unsteady; slip flow; free convection; chemical reaction

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1. Introduction

The applications of hydromagnetic viscous incompressible flow in science and engineering involving heat and mass transfer under the influence of chemical reaction is of great importance to many areas of science and engineering. This frequently occurs in petro-chemical industry, power and cooling systems, chemical vapour deposition on surfaces, cooling of nuclear reactors, heat exchanger design, forest fire dynamics and geophysics as well as in magneto hydrodynamic power generation systems. Chambre and Young [1] presented a first order chemical reaction in the neighbourhood of a horizontal plate. Das et al. [2] investigated the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past a vertical plate with a constant heat and mass transfer. Muthucumarswamy and Ganesan [3] studied the effect of chemical reaction and injection on the flow characteristics in an unsteady upward motion of an isothermal plate. Ibrahim and Makinde [4] presented a mathematical model for a two dimensional, steady, incompressible electrically conducting, laminar free convection boundary layer flow of a continuously moving vertical porous plate in a chemically reactive medium in the presence of a transverse magnetic field. Prakash et al. [5] have made investigation of fluid flow with chemical reaction on unsteady MHD mixed convective flow over a moving vertical porous plate.

Rarefaction effects must be considered in gases in which the molecular mean free path is comparable to the plate's characteristic domain. The continuum assumption is no longer valid and the gas exhibits non-continuum effects such as velocity slip, temperature jump and concentration jump. Traditional examples of non-continuum gas flows such as high altitude aircraft are vacuum technology. A study of vorticity of fluctuating flow of a visco-elastic fluid past an infinite plate with variable suction in slip flow regime was made by Mittal and Mukesh Bijalwan [6]. Rajesh Johari et al. [7] analyzed unsteady MHD flow through porous medium and heat transfer past a porous vertical moving plate with heat source. Free convection flow of magneto polar fluid through porous medium in slip flow regime with mass transfer was studied by Rajput et al. [8]. Dulal pal and Babulal Talukdar [9] reported perturbation analysis of unsteady magneto hydromagnetic convective heat and mass transfer in a boundary layer slip flow past a vertical permeable plate with thermal radiation and chemical reaction. Anjali Devi and Wilfred Samuel Raj [10] investigated the effects of thermo-diffusion on unsteady hydro-magnetic free convection flow of a viscous, incompressible, electrically conducting fluid with heat and mass transfer past moving porous vertical plate of infinite length with time dependent suction in the presence of heat source in a slip flow regime. In all the above mentioned investigations it is noted however that they have not considered slip flow due to jump in concentration which is practically important.

In this paper, it is proposed to study chemical reaction effects on unsteady hydrodynamic free convection flow past a moving vertical plate with time dependent suction in the presence of heat source in a slip flow regime with slip due to jump in temperature and concentration. Slip flow conditions for the velocity, jump in temperature and jump in concentration are taken into account in the boundary conditions. The dimensionless governing equations are solved using perturbation technique and the expressions for velocity, temperature and concentration are obtained. Non-dimensional velocity, temperature and concentration are discussed through graphs for different values of parameters entering into the problem.

2. Mathematical Formulation

The x^* axis is taken along the plate in the upward direction and y^* axis is normal to it. Due to semi-infinite plane surface assumptions, all the flow variables except pressure are functions of y^* and t^* only. A constant transverse magnetic field is applied in the direction of y^* axis. In the present work, the following assumptions are made: The flow is unsteady and laminar, and the magnetic field is applied perpendicularly to the plate. The fluid under consideration is viscous, incompressible and electrically conducting with constant physical properties. The magnetic Reynolds number is assumed to be small enough so that the induced magnetic field can be neglected. It is also assumed that there is no applied voltage, which implies the absence of an electric field. Slip flow regime is considered.

Under, these assumptions, the governing boundary layer equations of the problem are given by

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{1}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{dU_\infty^*}{dt^*} - \frac{\sigma B_o^2}{\rho}(u^* - U_\infty^*) + \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f (T^* - T_\infty^*)$$
$$+ g\beta_c (c^* - c_\infty^*) - \frac{\upsilon}{k^*}(u^* - U_\infty^*) \tag{2}$$

$$\rho C_p \left(\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} \right) = K \frac{\partial^2 T^*}{\partial y^{*2}} + S^*(T^* - T_\infty^*) \tag{3}$$

$$\frac{\partial c^*}{\partial t^*} + v^* \frac{\partial c^*}{\partial y^*} = D \frac{\partial^2 c^*}{\partial y^{*2}} + D_1 \frac{\partial^2 T^*}{\partial y^{*2}} - D_2 (c^* - c_\infty^*) \tag{4}$$

Where u^* and v^* are components of velocities along and perpendicular to the plate x^* and y^* are distance along and perpendicular to the plate respectively, ρ is the density of the fluid, g is the acceleration due to gravity, σ is the electrical conductivity, B_0 is Magnetic flux density, β_f is the coefficient of volume expansion of the working fluid, β_c is the coefficient of volumetric expansion with concentration, U_∞^* is the velocity of the fluid in the free stream, υ is the kinematic viscosity, T^* is the temperature of the fluid, T_∞^* is the temperature of the fluid in the free stream, K is thermal conductivity, S^* is coefficient of heat source, c^* is the concentration of the fluid, c_∞^* is concentration at infinity, D is chemical molecular diffusivity, D_1 the thermal diffusivity, D_2 the chemical reaction rate constant and C_P is the specific heat at constant pressure.

The slip flow boundary conditions are given by

$$u^* = u_w^* + h_1^* \frac{\partial u^*}{\partial y^*}; \quad T^* = T_w^* + h_2^* \frac{\partial T^*}{\partial y^*}; \quad c^* = c_w^* + h_3^* \frac{\partial c^*}{\partial y^*} \; at \; y^* = 0$$
$$u^* \to U_\infty^* = U_0(1 + \varepsilon e^{\delta^* t^*}); \quad T^* \to T_\infty^*; \quad c^* \to c_\infty^* \quad as \; y^* \to \infty \tag{5}$$

Where u_w^* the velocity at the wall, T_w^* is temperature at the wall, c_w^* is the concentration at the wall, ε and δ^* are scalar constants which are less than unity and $\varepsilon \ll 1$ and U_0 is the scale of stream velocity.

The plate is subjected to variable suction and from the equation of continuity, it can be written as

$$v^* = -V_0 (1 + \varepsilon \alpha e^{\delta^* t^*}) \tag{6}$$

Where α a real positive constant, ε and $\varepsilon\alpha$ are small less than unity, V_0 is the scale of the suction velocity which has a non-zero positive constant.

Introducing the following non dimensional scheme

$$u=\frac{u^*}{U_0},\quad v=\frac{v^*}{V_0},\quad t=\frac{t^*V_0^2}{\upsilon},\quad y=\frac{V_0y^*}{\upsilon},\quad \delta=\frac{\delta^*\upsilon}{V_0^2},\quad u_w=\frac{u_w^*}{U_0},\quad U=\frac{U_\infty^*}{U_0},\quad \theta=\frac{T^*-T_\infty^*}{T_w^*-T_\infty^*},$$

$$C=\frac{c^*-c_\infty^*}{c_w^\infty-c_\infty^*},\quad K=\frac{k^*V_0^2}{\upsilon^2},\quad Gr=\frac{\upsilon g\,\beta_f}{U_0V_0^2}(T_w^*-T_\infty^*),\quad \Pr=\frac{\mu C_p}{K},\quad Gm=\frac{\upsilon g\,\beta_c}{U_0V_0^2}(c_w^*-c_\infty^*),$$

$$M=\frac{\sigma B_0^2\upsilon}{\rho V_0^2},\quad S=\frac{S^*\upsilon^2}{KV_0^2},\quad Sc=\frac{\upsilon}{D},\quad So=\frac{D_1(T_w^*-T_\infty^*)}{\upsilon(C_w^*-C_\infty^*)},\quad Kr=\frac{D_2\upsilon}{V_0^2}$$

The governing equations of the problem in non dimensional form are given by

$$\frac{\partial u}{\partial t}-(1+\varepsilon\,\alpha\,e^{\delta t})\frac{\partial u}{\partial y}=\frac{dU}{dt}+\frac{\partial^2 u}{\partial y^2}+Gr\theta+GmC-M[u-U(t)]-\frac{u-U(t)}{k} \tag{7}$$

$$\frac{\partial \theta}{\partial t}-(1+\varepsilon\,\alpha\,e^{\delta t})\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2\theta}{\partial y^2}+\frac{S}{\Pr}\theta \tag{8}$$

$$\frac{\partial C}{\partial t}-(1+\varepsilon\,\alpha\,e^{\delta t})\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}+So\frac{\partial^2\theta}{\partial y^2}-KrC \tag{9}$$

Where *Gr* is the Grashof number, *Gm* is the solutal Grashof number, *Pr* is the Prandtl number, *M* is the magnetic field parameter, *Sc* is the Schmidt number, S is the heat source parameter, So is the thermo diffusion parameter and *Kr* is the chemical reaction parameter.

The corresponding boundary conditions in non dimensional form are

$$\begin{aligned} &u=u_w+h_1\frac{\partial u}{\partial y};\quad \theta=1+h_2\frac{\partial\theta}{\partial y};\quad C=1+h_3\frac{\partial C}{\partial y}\quad && at\quad y=0\\ &u\to U(t);\quad \theta\to 0;\quad C\to 0 && as\quad y\to\infty \end{aligned} \tag{10}$$

3. Solution of the Problem

In order to solve the nonlinear partial differential equations, the above systems of partial differential equations are reduced to a system of ordinary differential equations in a dimensionless form. The velocity, temperature and concentration are assumed in the following form:

$$\begin{aligned} u&=u_0(y)+\varepsilon\,e^{\delta t}u_1(y)+O(\varepsilon^2)\\ \theta&=\theta_0(y)+\varepsilon\,e^{\delta t}\theta_1(y)+O(\varepsilon^2)\\ C&=C_0(y)+\varepsilon\,e^{\delta t}C_1(y)+O(\varepsilon^2) \end{aligned} \tag{11}$$

By substituting (11) in equations (7) to (9), the following equations are obtained. The corresponding boundary conditions are also calculated by substituting in equation (10).

$$u_0'' + u_0' - \left(M + \frac{1}{k}\right)u_0 = -Gr\theta_0 - GmC_0 - \left(M + \frac{1}{k}\right) \tag{12}$$

$$u_1'' + u_1' - \left(M + \delta + \frac{1}{k}\right)u_1 = -Gr\theta_1 - GmC_1 - \alpha u_0' - \left(M + \delta + \frac{1}{k}\right) \tag{13}$$

$$\theta_0'' + \Pr\theta_0' + S\theta_0 = 0 \tag{14}$$

$$\theta_1'' + \Pr\theta_1' + (S - \Pr\delta)\theta_1 = -\alpha\Pr\theta_0' \tag{15}$$

$$C_0'' + ScC_0' - KrScC_0 = -ScSo\theta_0'' \tag{16}$$

$$C_1'' + ScC_1' - (Kr + \delta)ScC_1 = -ScSo\theta_1'' - \alpha ScC_o' \tag{17}$$

With the corresponding boundary conditions

$$\begin{aligned} &u_0 = u_w + h_1\frac{\partial u_0}{\partial y};\, u_1 = h_1\frac{\partial u_1}{\partial y};\, \theta_0 = 1 + h_2\frac{\partial\theta_0}{\partial y};\, \theta_1 = h_2\frac{\partial\theta_1}{\partial y}; \\ &C_0 = 1 + h_3\frac{\partial C_0}{\partial y};\, C_1 = h_3\frac{\partial C_1}{\partial y} \qquad at\ y = 0 \\ &u_0 \to 1;\, u_1 \to 1; \quad \theta_0 \to 0;\, \theta_1 \to 0; \quad C_0 \to 0;\, C_1 \to 0 \quad as\ y \to \infty \end{aligned} \tag{18}$$

Where u_w is the velocity ratio parameter, h_1 is the slip parameter due to velocity, h_2 is the slip parameter due to jump in temperature and h_3 is the slip parameter due to jump in concentration.

On solving (12) to (17) subject to boundary conditions in (18) the solutions are given by

$$\theta_0 = B_1 e^{-m_1 y} \tag{19}$$

$$\theta_1 = B_2 e^{-m_1 y} + B_3 e^{-m_2 y} \tag{20}$$

$$C_0 = B_4 e^{-m_1 y} + B_5 e^{-m_3 y} \tag{21}$$

$$C_1 = B_6 e^{-m_1 y} + B_7 e^{-m_2 y} + B_8 e^{-m_3 y} + B_9 e^{-m_4 y} \tag{22}$$

$$u_0 = 1 + B_{10} e^{-m_1 y} + B_{11} e^{-m_3 y} + B_{12} e^{-m_5 y} \tag{23}$$

$$u_1 = B_{13} e^{-m_1 y} + B_{14} e^{-m_2 y} + B_{15} e^{-m_3 y} + B_{16} e^{-m_4 y} + B_{17} e^{-m_5 y} + B_{18} e^{-m_6 y} + 1 \tag{24}$$

4. Result and Discussion

In the present work, we have analyzed flow, heat and mass transfer on unsteady hydromagnetic free convective flow of an electrically conducting fluid past a semi-infinite moving porous vertical plate with thermodiffusion and chemical reaction effects in slip flow regime. Final results are computed for variety of physical parameters which are presented by means of graphs. The results are obtained to illustrate the influence of the thermal Grashof number Gr, the solutal Grashof number Gm, Prandtl number Pr, magnetic field parameter M, Schmidt number Sc, heat source parameter S , thermo diffusion parameter So and chemical reaction parameter Kr on the velocity, temperature and the concentration profiles. While the values of some of the physical parameters are taken as constant such as $Sc = 0.22$, $So = 1$, $S = 0.1$, $Gr = 1$, $Gm = 4$, $K = 1$, $h_1 = 0.4$, $h_2 = 0.5$, $t = 1$, $\delta = 0.1$, $\varepsilon = 0.01$, $\alpha = 1$ in all the figures. We have extracted interesting insights regarding the influence of all the parameters that govern this problem. The influence of parameters on horizontal velocity, temperature and concentration profiles can be analyzed from Figs. 1-7.

Fig.1 depicts the effect of velocity ratio parameter over the velocity profiles. The effect of velocity ratio parameter is to accelerate the velocity and its influence is highly dominant near the plate whereas it remains uniform as we move far away from the plate. The effect of magnetic field on velocity profiles in the boundary layer is depicted in Fig.2. It is interesting to note that the effect of magnetic field is to decrease the value of the velocity profile throughout the boundary layer. Because the presence of magnetic field in an electrically conducting fluid introduces a force called the Lorentz force, which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. This type of resisting force slows down the fluid velocity as shown in this figure.

The effect of chemical reaction parameter is highlighted in Fig. 3. The increase of the chemical reaction parameter Kr leads to decrease the velocity profiles. The effect of Prandtl number Pr on the temperature profiles is shown in Fig. 4. It is seen that the increase in the Prandtl number leads to fall in the temperature of the fluid. The reason is that lower Pr value has more uniform temperature distribution across the thermal boundary layer as compared to higher Pr value. This phenomenon occurs when the lesser values of Prandtl number are equivalent to increasing thermal conductivity. Therefore, heat is capable to diffuse away from the heated surface more quickly compare to bigger values of Prandtl number. The effect of the reaction rate parameter Kr on the concentration profiles for generative chemical reaction is shown in Fig. 5. It is observed that increasing the value of the chemical reaction decreases the concentration of species in the boundary layer, this is due to the fact that destructive chemical reduces the solutal boundary layer thickness and increases the mass transfer. The impact of slip parameter due to jump in concentration over the concentration profiles is noted in Fig. 6. The concentration of the flow field is decreases with increase in slip parameter due to jump in concentration.

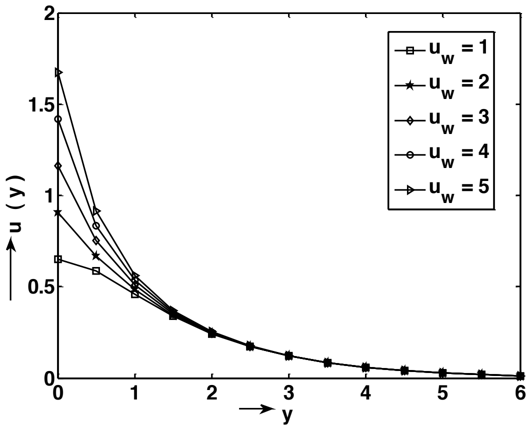

Fig. 1. Effect of u_w over the Velocity profiles when Pr = 0.71, Kr = 1, M = 2, h_3 = 1.

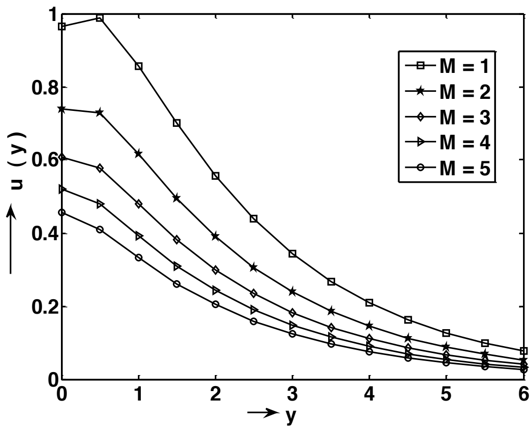

Fig. 2. Velocity profiles for various values of M when Pr = 0.71, Kr = 1, h_3 = 1, u_w = 1.

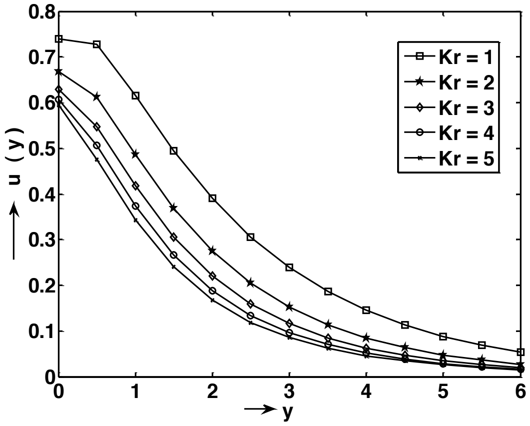

Fig. 3. Velocity profiles for various values of Kr when M = 2, Pr = 0.71, $u_w = 1$, $h_3 = 1$.

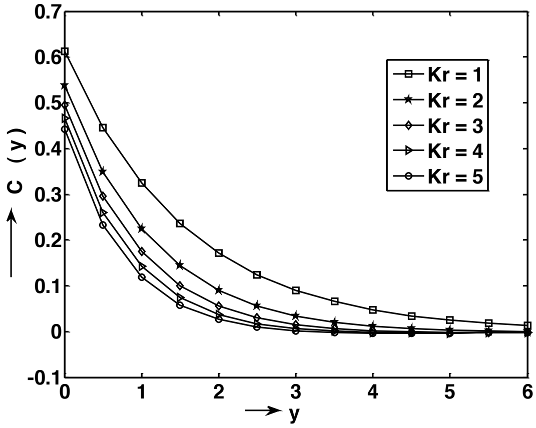

Fig. 5. Concentration profiles for different values of Kr

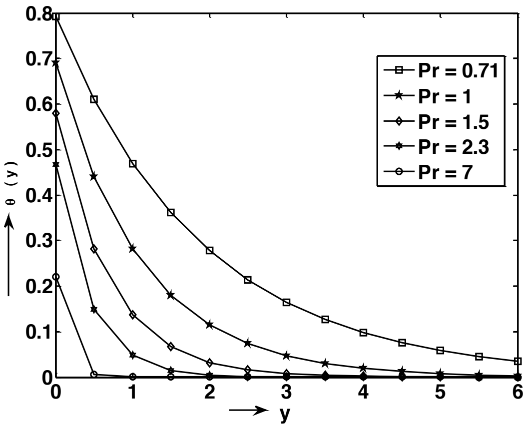

Fig. 4. Effect of Prandtl number over the Temperature distribution

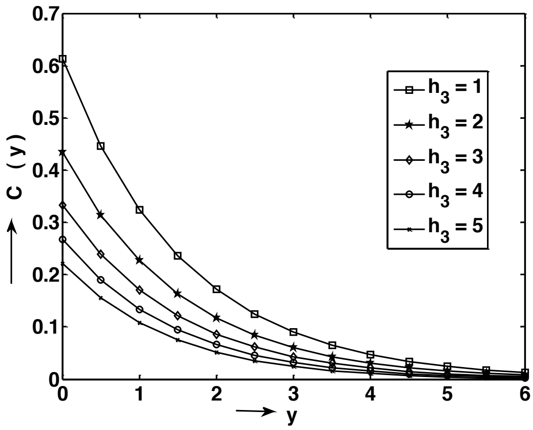

Fig. 6. Effect of Slip parameter h_3 over the Concentration distribution

5. Conclusions

In this study, the non-linear and coupled governing equations are solved analytically by perturbation technique. Velocity, temperature and concentration profiles are presented graphically and analyzed. The fundamental parameters found to effect the problem under consideration are the chemical reaction parameter, magnetic field parameter, Prandtl number, velocity ratio parameter and slip parameter due to jump in concentration. It is found that the velocity decreases due to the effects of magnetic field parameter and chemical reaction parameter whereas the velocity is increased by velocity ratio parameter. The temperature of the plate decreases with increasing the values of the Prandtl number. Additionally, the concentration is decreased in the presence of chemical reaction and slip parameter due to jump in concentration. In the absence of chemical reaction and slip parameter due to jump in concentration, these results are in good agreement with the results of Anjali Devi and Wilfred Samuel Raj [10].

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